Exploring Performance Metrics of John Conway's Game of Life

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Abstract

This project explores the performance metrics of John Conway's Game of Life, a classic cellular automaton known for its emergent complexity. The primary objectives include analyzing the survival rates of various initial configurations and studying the population dynamics over successive generations. The Game of Life operates on a grid of cells, governed by simple rules of birth, death, and survival based on neighboring cell states. By running simulations with random initial setups, we aim to quantify the resilience of different patterns and understand the growth trends of populations within the game. Our findings reveal an optimal population density critical for stable configurations and identify the lognormal distribution as the best fit for prolonged survival at random initial probabilities. These insights contribute to the understanding of complex systems under deterministic rules.

1 Background

1.1 Introduction

John Conway's Game of Life is a cellular automaton devised by the British mathematician John Horton Conway in 1970. It is a zero-player game, meaning that its evolution is determined by its initial state, requiring no further input. The Game of Life has gained widespread recognition for its ability to simulate complex patterns and behaviors from simple initial configurations and rules. [6]

1.2 How it Works

The game is played on a grid of square cells, each of which can be in one of two states: alive or dead. The state of each cell in the next generation is determined by a set of rules that consider the current state of the cell and the states of its eight neighbors (adjacent cells, including diagonals). The rules are as follows:

- Birth: A dead cell with exactly three live neighbors becomes a live cell.
- Survival: A live cell with two or three live neighbors stays alive.
- Death: In all other cases, a cell dies or remains dead.

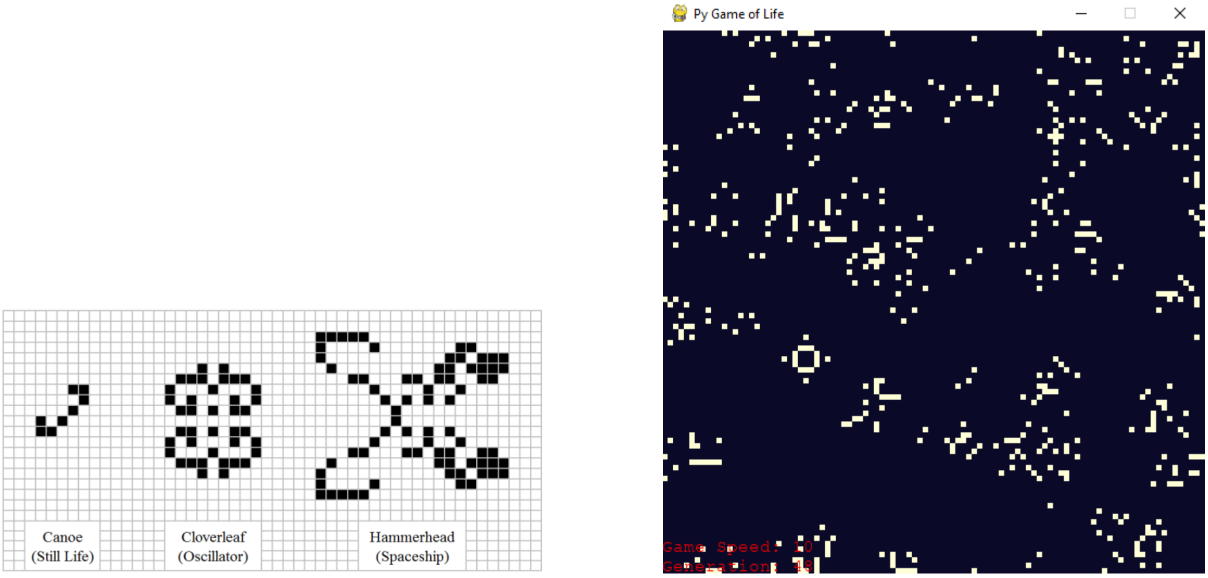
These simple rules lead to a wide variety of behaviors, from stable structures and repeating patterns to seemingly chaotic evolutions.

1.3 Characteristics of the Game Rules

The Game of Life showcases several key characteristics. Firstly, it operates in a totally deterministic nature, meaning that despite the appearance of randomness in its grid evolution, if provided with the same initial configuration, the sequence of generations will consistently unfold in the same manner. This deterministic quality underlines the predictability within its seemingly chaotic behavior.

Secondly, the game demonstrates emergent complexity, where simple starting configurations can give rise to intricate and often unexpected structures. This phenomenon highlights the capacity of local interactions among cells to generate complex, global behaviors over successive generations.

Lastly, the Game of Life exhibits a wide spectrum of patterns, showcasing pattern diversity within its grid. These patterns include static formations known as still lifes, repetitive structures called oscillators



((a)) Example Patterns [8] ((b)) Game of Life Implementation Screenshot

Figure 1: Comparison of Game of Life patterns and implementation

that cycle after a fixed number of generations, and mobile configurations termed spaceships that traverse the grid. The combination of these patterns provide a seemingly limitless possibility of outcomes possible within the game's simple framework.

1.4 Importance and Applications

The Game of Life is more than a mathematical curiosity; it serves as a model for various processes in physics, biology, and computer science. It illustrates principles of self-organization and complexity and has applications in fields such as theoretical biology, where it helps model population dynamics and other biological systems.

2 The Problem at Hand

In this project, we aim to explore two primary performance metrics of the Game of Life: survival rate and population dynamics. The survival rate will measure how long random initial configurations can sustain life over successive generations. Population dynamics will analyze the changes in the number of live cells over time, providing insights into the growth patterns and stability of various configurations. By examining these metrics, we seek to understand the underlying factors that contribute to the resilience and behavior of patterns in the Game of Life.

3 Main Findings

3.1 Simulation Setup

To create the simulation, we programmed the Game of Life in Python, utilizing NumPy for efficient array processing and Pygame for visualizing and displaying the cell states. The simulation departs from the traditional game, which has an infinite grid, instead we employed a toroidal array, where the edges are connected. This means that cells on the far-right edge are next to cells on the far-left edge, and similarly for the top and bottom edges. This setup ensures continuity and avoids edge effects that can otherwise distort the results.

Each simulation is initialized as a grid of 100x100 cells, resulting in a total of 10,000 cells. The initial state of each cell is determined by a probability parameter, which dictates whether a cell starts as alive or dead. We experiment with varying this probability to investigate its impact on population dynamics. For our simulation we will define a steady state being achieved when all cells are **dead**, **oscillators**, or **still lifes**.

3.2 Simulation Execution

Initialization:

Set up a 100x100 grid.

Initialize each cell based on a specified probability of being alive.

Running the Simulation:

Update the grid according to the rules of the Game of Life:

Birth: A dead cell with exactly three live neighbors becomes a live cell.

Survival: A live cell with two or three live neighbors stays alive.

Death: In all other cases, a cell dies or remains dead.

Utilize a toroidal array to handle edge cases, where the grid wraps around on itself.

Measurement and Data Collection:

Track the number of live cells (population size) at each generation.

Record the total number of generations until the simulation ends (either all cells are dead, or a steady state is reached).

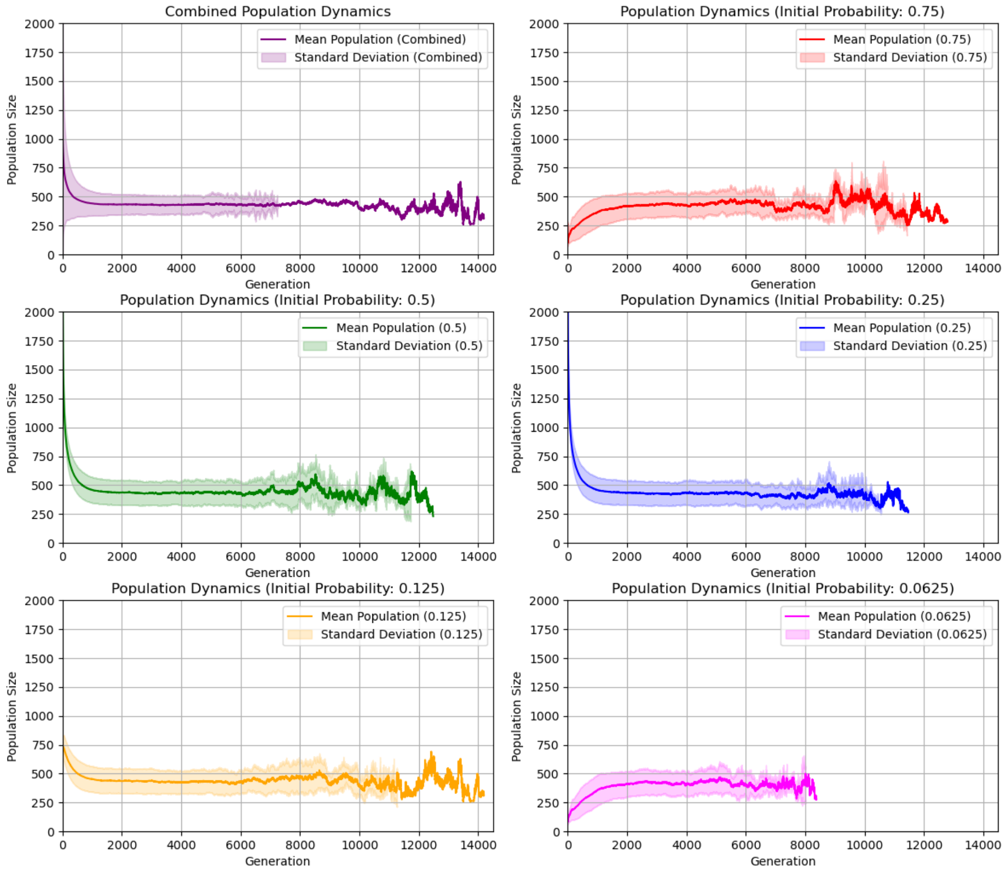


Figure 2: Population Dynamics

4 Results

We ran each five sets of simulations, each with a different parameter for the initial cell living probability, ranging from a probability of just 0.0625 up to 0.75. Each of these five sets consisted of 5,000 games run until steady state, for a total of 25,000 games simulated.

4.1 Population Dynamics

The population dynamics depicted in Figure 2 reveal several patterns. This figure shows both the aggregated populations and those originating from each initial probability. A clear trend can be observed, as we progress in number of generations survived, each simulation converges to a similar population level, suggesting that this optimal population density is crucial for sustaining a stable Game of Life under our specific parameters. From the aggregated statistics plot we can pinpoint a stabilization point at slightly under 500 living cells, corresponding to a population density of 0.05.

Intriguingly, simulations commencing with a probability of 0.75 exhibit an immediate population crash, due to overpopulation, followed by a recovery to the stable population level. Similarly, simulations initiated with probabilities of 0.5 and 0.25 experienced rapid population declines converging to the same stable state.

4.2 Survival Rate: Stable Population

We can also gain insight into the Game of Life's survival rate as presented in Figure 3. This figure showcases the kernel density estimate (KDE) plots corresponding to each initial probability, offering a visual representation akin to a histogram. Accompanying these KDE plots are the box-plots, both of which unveil a discernible trend: intermediate probabilities (0.125, 0.25, 0.5) exhibit a prolonged survival, more generations, compared to outer probabilities (0.0625, 0.75). We attribute this trend to their proximity to the most stable population level of approximately 500 living cells, with a density of 0.05, as we discovered above in Figure 2.

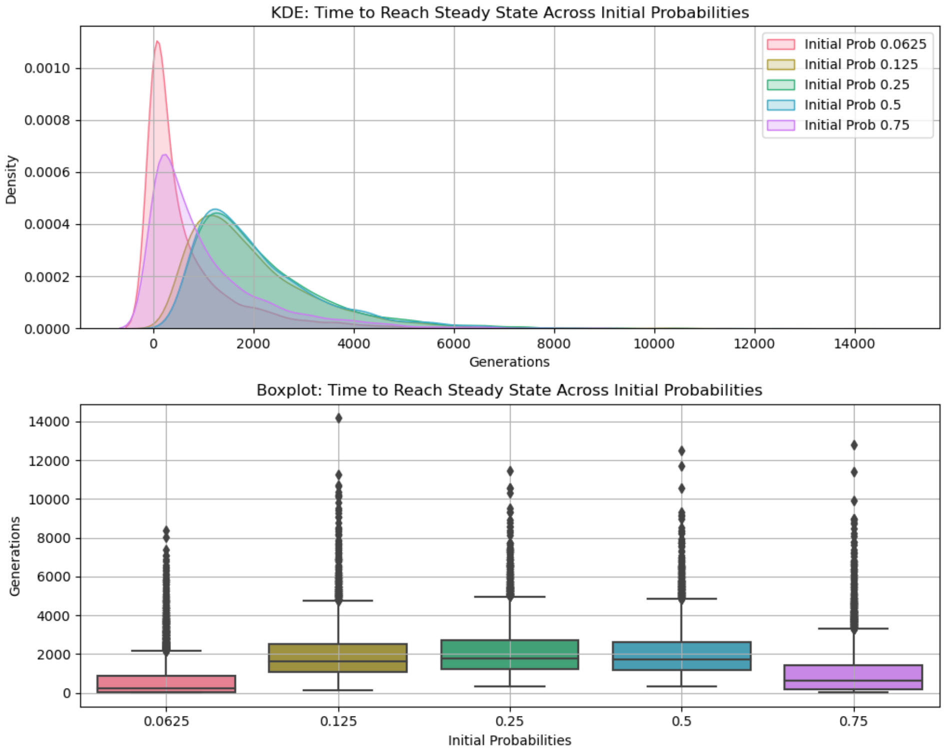


Figure 3: KDE Boxplot

4.3 Survival Rate: Distribution

In order to gain further insight into this survival rate phenomenon, we narrowed our focus solely to the starting probability 0.5. This allowed us to shift our objective towards identifying the best fitting distribution. To accomplish this, we assessed several distributions, including the following:

- A normal continuous random distribution: $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- An exponential continuous random distribution: $f(x) = \lambda e^{-\lambda x} \quad \text{for } x \geq 0$
- A lognormal continuous random distribution: $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}} \quad \text{for } x > 0$

- A gamma continuous random distribution: $f(x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)}$ for $x \geq 0$

- A beta continuous random distribution: $f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ for $0 \leq x \leq 1$

- A Weibull minimum continuous random distribution: $f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}$ for $x \geq 0$

- And Pearson type III continuous random distribution: $f(x) = \frac{|\beta|}{\Gamma(\alpha)}(x-\gamma)^{\alpha-1}e^{-\beta(x-\gamma)}$ for $x > \gamma$

For each of these, we fitted our data to the distribution and computed the SSE (Sum of Squared Errors). The best fitting distribution, lowest SSE, was the lognormal continuous random distribution. Subsequently, we generated plots encompassing all distributions as well as highlighting the top three best fitting distributions, as depicted in Figure 4. From this figure we can visually confirm that the lognormal continuous random distribution is indeed the closest match to our set of simulations starting with probability 0.5.

In our final figure, Figure 5, we've plotted the histogram of our ending generations along with the lognormal continuous random distribution and the Cumulative Distribution Function (CDF) of both, again we can observe that we can observe that out data closely fits this distribution.

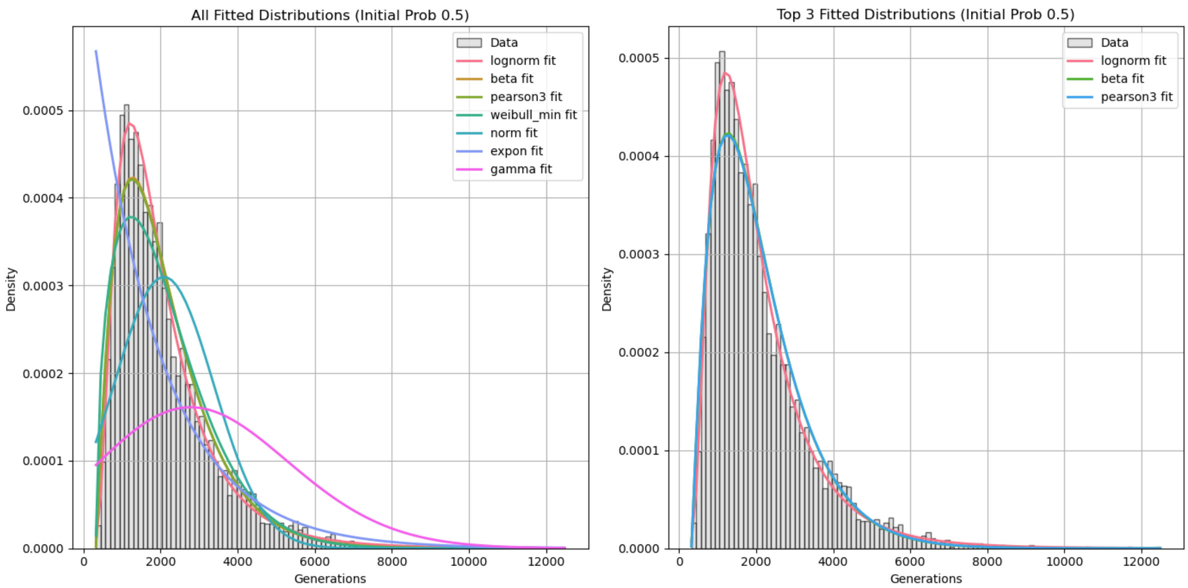


Figure 4: Fitted Distributions

5 Conclusion

In this project, we investigated the performance metrics of John Conway's Game of Life, focusing on survival rates and population dynamics across various initial configurations. Our simulations revealed an optimal population density, around 0.05 or approximately 500 living cells, as critical for achieving stable configurations. This stability was consistently observed across different initial probabilities, with intermediate probabilities (0.125, 0.25, 0.5) demonstrating prolonged survival times compared to the extremes (0.0625, 0.75).

The analysis of kernel density estimates (KDE) and box plots highlighted the resilience of configurations starting with our intermediate probabilities, further reinforcing the importance of initial population density in determining long-term stability. By narrowing our focus to the initial probability of 0.5, we conducted a distribution fitting analysis and identified the lognormal distribution as the best match for our data.

These findings contribute to a deeper understanding of the emergent behaviors in cellular automata and offer insights into the underlying mechanisms that govern these complex systems. The deterministic

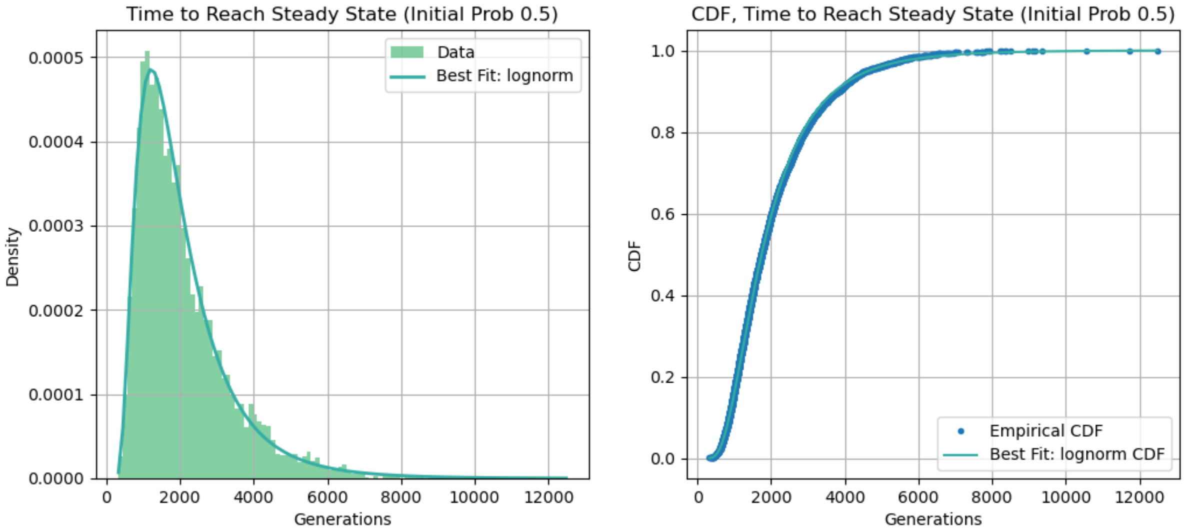


Figure 5: Histogram and CDF

nature of the Game of Life, coupled with its ability to produce intricate and diverse patterns, underscores its value as a model for studying self-organization and complexity in various scientific domains.

5.1 Future Work

We can envision many ways that this work could be continued or expanded upon to further develop this project's findings. Future research could explore different grid sizes and different boundary conditions to investigate if the resulting optimal population density and distribution are universal or only apply to these specific conditions. It would also be possible to study differing rule sets with regard to cell birth, survival, and death to further understand the dynamics of this computational system. Furthermore, this project could be replicated, starting with the same initial state but modifying the computational rules of the game to be probabilistic [1]. Lastly, this project could be replicated in three dimensional space or higher [3] [4] exploring the change in survival rate and population with the added dimensions.

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